Matrices in composite Hilbert spaces.

We use a basis consisting of separable (product) vectors, and we order them "lexicographically"

$|0\rangle = |0\rangle|0\rangle$

$|1\rangle = |0\rangle|1\rangle$

$|2\rangle = |1\rangle|0\rangle$

$|3\rangle = |1\rangle|1\rangle$

So rows and columns get <u>pairs</u> of indices

$$\begin{array}{c} \\ 00 \\ 01 \\ 10 \\ 11 \end{array}\begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \end{array}$$

$\rho_{11;01}$ sits here

For matrix elements, use

$\rho^{i\alpha}{}_{j\beta}$

or

$\rho_{i\alpha;j\beta}$

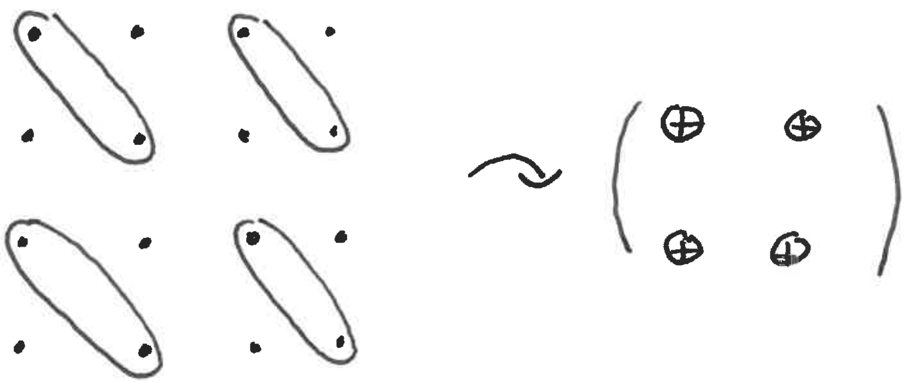
Partial traces:

$$(\mathrm{Tr}_2\, \rho)^i{}_j = \sum_\alpha \rho^{i\alpha}{}_{j\alpha} = \sum_\alpha \rho_{i\alpha;j\alpha}$$

$$(\mathrm{Tr}_1 \rho)^\alpha{}_\beta = \sum_i \rho^{i\alpha}{}_{i\beta} = \sum_i \rho_{i\alpha;i\beta}$$

These are operators acting on the factors

Tr_2 :



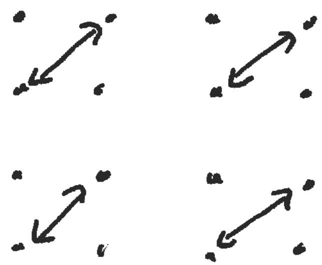
Tr_1 :

Partial transpose:

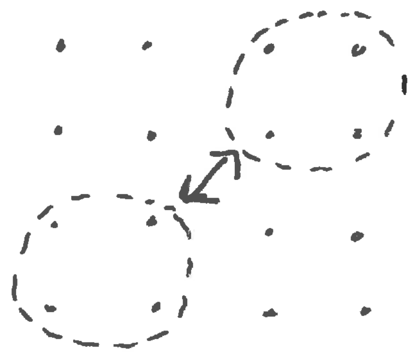
$$(\rho^{T_2})_{i\alpha;j\beta} = \rho_{i\beta;j\alpha}$$

$$(\rho^{T_1})_{i\alpha;j\beta} = \rho_{j\alpha;i\beta}$$

ρ^{T_2}:



$\rho^{T_1} =$



You can also think of the matrix as a "matrix of matrices"

$$M = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$$

where A, B, C, D är 2×2 (or $N \times N$!) matrices

$\mathcal{H}_2 \otimes \mathcal{H}_2$ $\quad$ $\mathcal{H}_2 \otimes \mathcal{H}_N$

$$\mathrm{Tr}_2 M = \begin{pmatrix} \mathrm{Tr}\,A & \mathrm{Tr}\,B \\ \mathrm{Tr}\,C & \mathrm{Tr}\,D \end{pmatrix}$$

$$M^{T_2} = \begin{pmatrix} A^T & B^T \\ C^T & D^T \end{pmatrix}$$

$$\mathrm{Tr}_1 M = A + D$$

$$M^{T_1} = \begin{pmatrix} A & C \\ B & D \end{pmatrix}$$

$$\mathrm{Tr}_2 \mathrm{Tr}_1 M = \mathrm{Tr}_1 \mathrm{Tr}_2 M = \mathrm{Tr}\,M = \mathrm{Tr}\,A + \mathrm{Tr}\,D$$

$$\left(M^{T_2}\right)^{T_1} = M^T$$

<u>Note added</u> (for my own conscience)!

- In the relativity course you learn that it is important to distinguish upper and lower indices (sometimes!).
- I don't seem to be doing it consistently in the notes!
- Admitted. What happens is that complex conjugation interchanges upstairs and downstairs (and "transpose" is the complex conjugate of "dagger").
- <u>Let's not worry about it</u>!