FK8024 Special Topics in Theoretical Physics

Problem 4.5

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The task

We want to show that we, if we at will are able to move massive particles between ground state and exited states by way of photons, either will be able to build a perpetual motion machine, or that photons are affected by gravity.

Designing the machine

We start by constructing a machine with buckets on a vertical conveyor belt. We let these buckets hold one identical massive particle each. The ones on the right will contain excited particles, and the ones on the left will contain particles in their ground state. This will cause the machine to rotate, as the right hand side is heavier than the left hand side. When a particle reaches the bottom of the construction, it will be made to send out a photon, which, by way of our perfect mirrors, will be directed to, and if possible received by, the ground state particle that will have just reached the top of the construction. An example of a design can be seen in figure 1, on the right. Since this will happen every time a new particle reaches the bottom of our construction, we can conclude that we have built a perpetual motion machine, unless the photons somehow lose energy along the way.

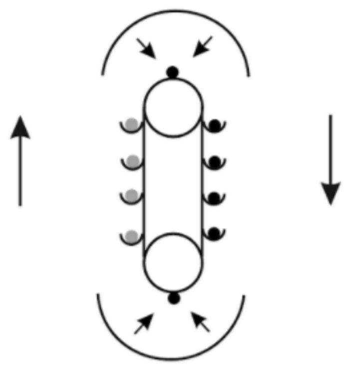

Figure 1: Our perpetual bucket machine. The black particles on the right are excited.

Photon energy

To see if the photons will lose energy, we use that ∂_t is a Killing vector and that

$$k_a \xi^a\big|_{p_1} = k_a \xi^a\big|_{p_2}. \tag{1}$$

We also use that

$$\omega_1 = \frac{k_a \xi^a}{(\xi_b \xi^b)^{1/2}}\bigg|_{p_1} \quad \text{and} \quad \omega_2 = \frac{k_a \xi^a}{(\xi_b \xi^b)^{1/2}}\bigg|_{p_2}. \tag{2}$$

This will give us

$$\frac{\omega_1}{\omega_2} = \frac{\left(1 - \frac{2m}{r_2}\right)^{1/2}}{\left(1 - \frac{2m}{r_1}\right)^{1/2}}. \tag{3}$$

More details on the calculations can be found in the solution to problem 1.2. Finally, we use the relation between the frequency and the wavelength and that $m << r$, to rewrite our expression in the form

$$\lambda\omega = 2\pi c \quad \Rightarrow \quad \frac{\lambda_2}{\lambda_1} = 1 - \frac{m}{r_2} + \frac{m}{r_1} > 1. \tag{4}$$

Conclusions

These calculations give at hand that the energy of the photon will have been reduced when it reaches its destination particle compared to when it was emitted. This reduction in energy will make the photon unable to excite the intended atom and therefore the machine will stop moving quite quickly. If the system had been classical, the energy would have bleeded out slowly and the machine come to a halt more slowly. Had the gravitational field not redshifted the photons, the machine would have continued perpetually.