

Kit nr 2: Teleportation

Student Manual

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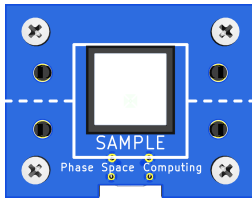
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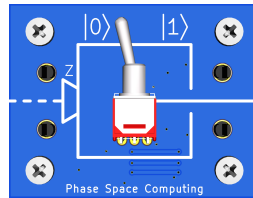
This lab kit contains material to demonstrate quantum teleportation, including entanglement source and analyzer.

You should read through the manual before starting the laboratory work.

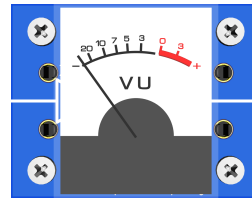
1 Content of the kit



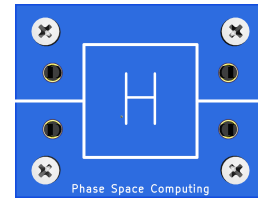
Sample button ×1: Re-samples the random numbers used in the simulation.



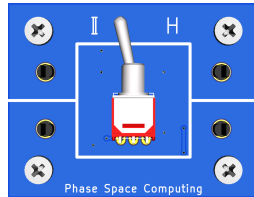
Source ×3: component that sends a qubit in one of the computational states $|0\rangle$ or $|1\rangle$ depending on the setting of the switch.



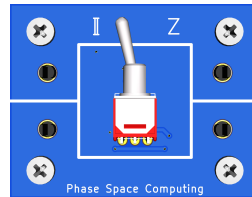
Measurement device ×3: Measures the quantum state and displays the result. The reading is $|0\rangle$ if the meter points to the left, and $|1\rangle$ if the meter points to the right (into the red area).



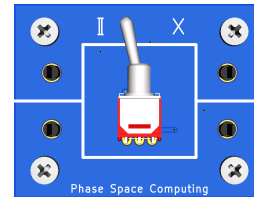
Hadamard gate ×2: Transforms states in the $|0\rangle/|1\rangle$ basis to the $|+\rangle/|-\rangle$ basis and vice versa.



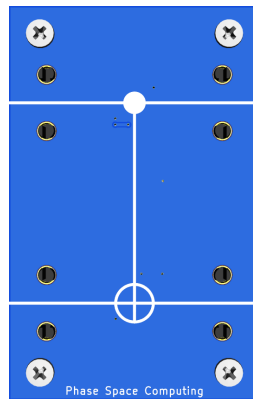
Classical-control Hadamard gate ×2: If enabled, transforms states in the $|0\rangle/|1\rangle$ basis to the $|+\rangle/|-\rangle$ basis and vice versa.



Classical-control Pauli-Z gate ×1: If enabled, inverts the states of the phase basis, $|+\rangle \leftrightarrow |-\rangle$.



Classical-control Pauli-X gate ×1: If enabled, inverts the states of the computational basis, $|0\rangle \leftrightarrow |1\rangle$.



CNOT gate (or Controlled NOT gate) ×2: the gate operates on two qubits. The second qubit (that goes in the lower part of the component) is called target qubit and is flipped by the gate if and only if the first qubit (the control qubit) is $|1\rangle$.

Power cord ×1

Patch cable (7 cm) ×10

Patch cable (30 cm) ×5

Patch cable (50 cm) ×2

2 Required reading on Quantum Teleportation

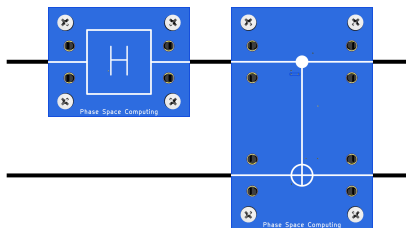
Read Sections 1.3, and 2.1–2.3 of M. A. Nielsen and I. L. Chuang (2010). *Quantum Computation and Quantum Information*. Vol. 10th Anniversary Edition. Cambridge University Press. ISBN: 978-1-107-00217-3.

3 Preparatory exercises

1. Verify that the Bell states constitute a orthonormal basis for the two-qubit state space,

$$\begin{aligned}
 |\psi_{00}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \\
 |\psi_{01}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle), \\
 |\psi_{10}\rangle &= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle), \\
 |\psi_{11}\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).
 \end{aligned} \tag{1}$$

2. Write down the Bell basis in the computational basis $|0\rangle/|1\rangle$ and in the phase basis $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. What conclusions can you draw concerning the correlation between individual measurement outcomes for the different states,
 - (a) when using the computational basis, and
 - (b) when using the phase basis?
3. Rewrite the Bell basis so that you can draw conclusions concerning the correlation between individual measurement outcomes when using the computational basis for one qubit and the phase basis for the other.
4. For the Bell basis, what are the distributions for single-system outcomes, in the computational and phase basis?
5. Calculate the output from the following gate array for all input states in the computational basis. What set of states does it generate?



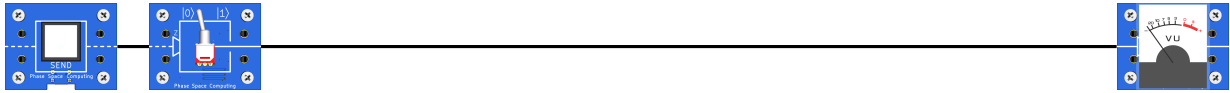
6. Reverse the gate array of Exercise 5. Show that measuring in the computational basis after your new array will give you a *Bell-state analyzer*, that can tell you which of the four Bell states that are input to the gate array.
7. For the Bell-state analyzer of Exercise 6, calculate the output probabilities for the input states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$, $|++\rangle$, $|+-\rangle$, $|-+\rangle$, and $|--\rangle$
8. What happens to the Bell states if you perform a Pauli-Z on the first qubit? What happens if you perform a Pauli-X on the first qubit? What happens if you do both?
- *9. Prove that a Bell state cannot be written as a tensor product $|\phi_A\rangle \otimes |\phi_B\rangle$.

4 Laboration tasks

From here, the laboratory work begins. A standard session should cover the tasks up to and including *Quantum Teleportation*.

Task 1 (Quantum bits): Verify that source and measurement works

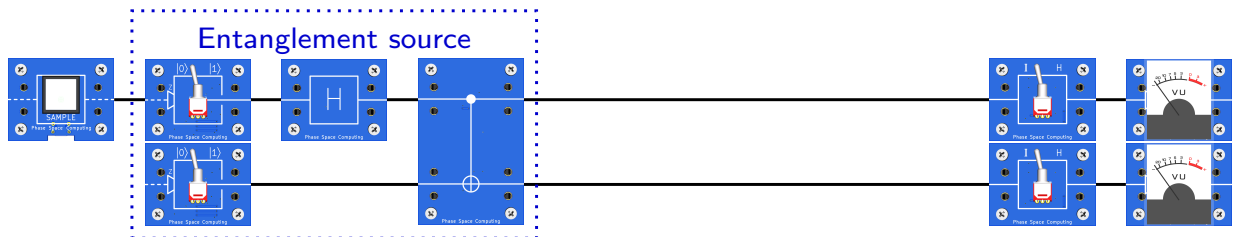
Materials: Power unit, send button, one source, one detector, one short and one long cable



Connect the source to the detector using the long connector cable, connect the sample button, connect power, and test the circuit by flipping the switch and pressing the sample button. When sending a 0, the measurement device should show a low value (the meter should point to the left), and when sending a 1, the measurement device should show a high value (the meter should point to the right, into the red area)

Task 2 (Quantum entanglement): Verify the correlations from an entanglement source

Materials: Power unit, sample button, two sources, two detectors, three Hadamards, one CNOT, cables

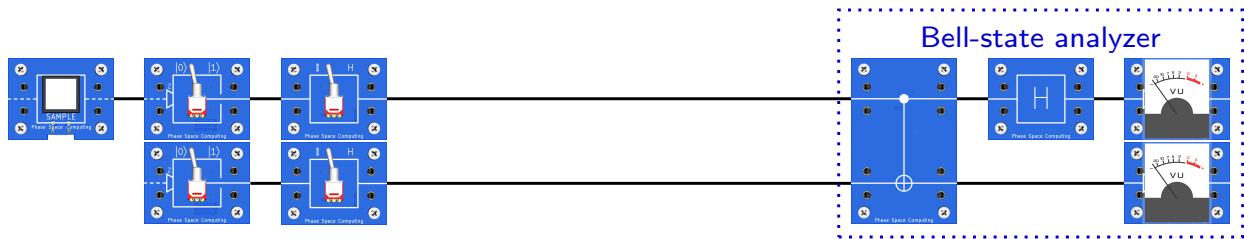


Connect as in the above picture. Preparatory exercise 5 should tell you what state is output from the entanglement source for the different settings of the internal qubit state sources. Exercise 2 tells you what correlations to expect for each state, for each setting of the pair of classically controlled Hadamards at the detectors. Some choices should give correlations, others should not.

- Verify that these correlations hold (there are four possible choices in the source, and four at the measurements, some combinations are more interesting than others).

Task 3 (Quantum entanglement): Verify that a Bell-state analyzer detects correlations between the qubits

Materials: Power unit, sample button, two sources, two detectors, three Hadamards, one CNOT, cables

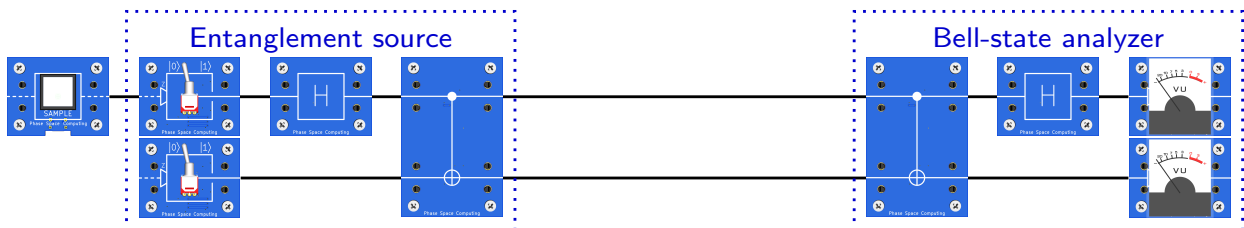


Connect as in the above picture. You can now use the qubit sources to generate a state with a correlation of your choice. Preparatory exercise 6 tells you what the possible output states are, and exercise 7 tells you the exact probability distribution, for each input state.

- ▶ Verify that the Bell-state analyzer finds states that have the correlation you have chosen. There are again eight different choices, four possible two-qubit states in four basis combinations.
- * Verify that the Bell-state analyzer gives the correct output probability for *one* input state in *either* the computational basis *or* the phase basis. This more formal verification requires a hypothesis test, and will take time. Choose the significance of the test wisely.

Task 4 (Quantum entanglement): Verify that entanglement source and Bell-state analyzer work

Materials: Power unit, sample button, two sources, two detectors, three Hadamards, two CNOTs, cables

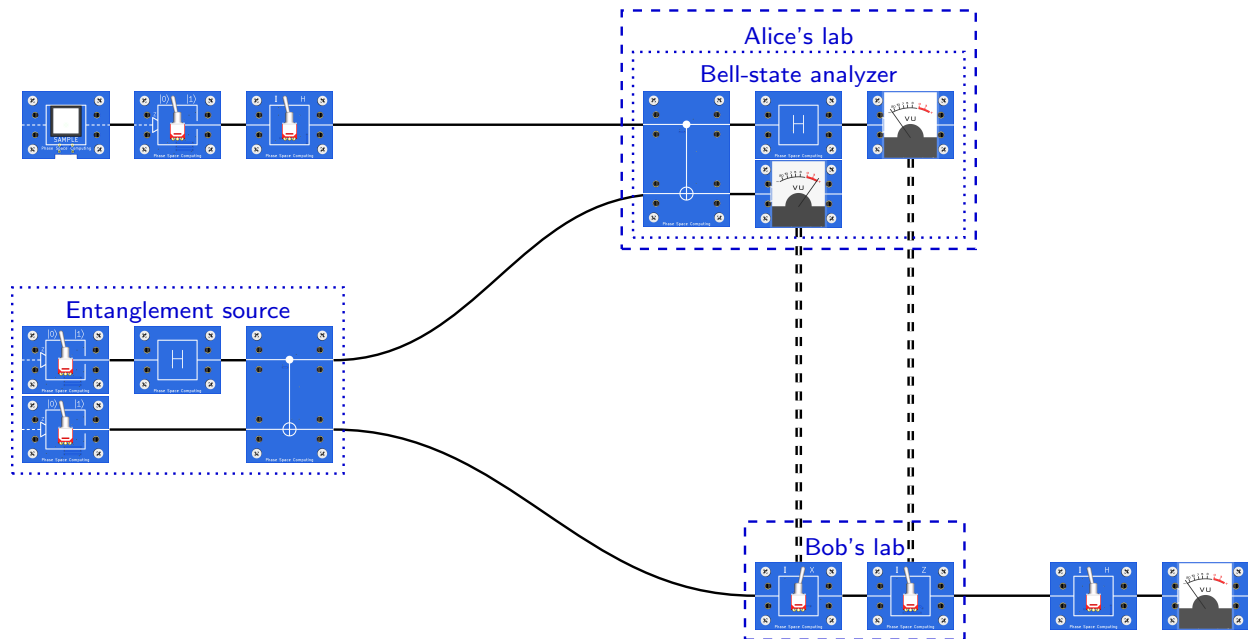


Connect as in the above picture. You can now use the qubit sources to generate a Bell state of your choice as in 5.

- ▶ Verify that the Bell-state analyzer finds the state you chose. Check all four combinations.

Task 5 Perform Quantum Teleportation

Material: Power unit, sample button, Source $\times 3$, Measurement $\times 3$, Hadamard gate $\times 2$, Classical-control Hadamard gate $\times 2$, CNOT gate $\times 2$, Classical-control Pauli-X gate $\times 1$, Classical-control Pauli-Z gate $\times 1$, cables



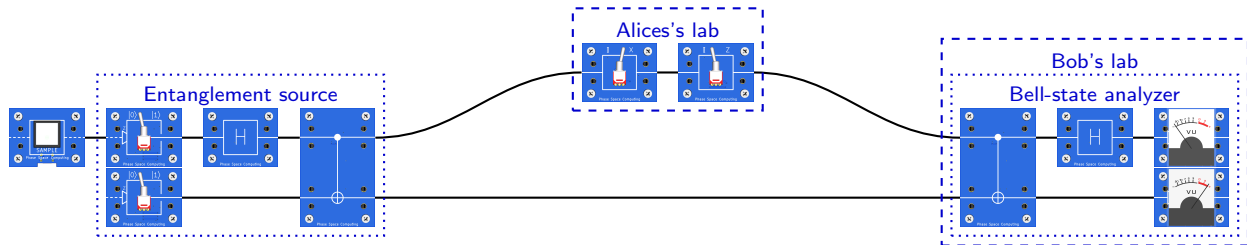
Decide who is going to be Alice and who is going to be Bob during this task. If possible, place a shielding between Alice and Bob so that they cannot see each other. The double lines connecting measurement devices with on/off switches denote classical communication between Alice and Bob.

► Perform quantum teleportation:

1. Have Alice choose a quantum state to teleport, one of $|0\rangle$, $|1\rangle$, $|+\rangle$ or $|-\rangle$
2. Have Alice read off the results in her lab and communicate them to Bob.
 - (a) If the lower measurement instrument outputs 1, X should be on in Bob's lab, otherwise off.
 - (b) If the upper measurement instrument outputs 1, Z should be on in Bob's lab, otherwise off.
3. Confirm that the output state in Bob's lab is the correct state. Choose the same basis as Alice and read off the output.
4. Repeat using different states

Task *6 Perform Superdense Coding

Material: Power unit, sample button, Source $\times 2$, Measurement $\times 2$, Hadamard gate $\times 2$, CNOT gate $\times 2$, Classical-control Pauli-X gate $\times 1$, Classical-control Pauli-Z gate $\times 1$, cables



Decide who is going to be Alice and who is going to be Bob during this task. If possible, place a shielding between Alice and Bob so that they cannot see each other.

► Perform quantum superdense coding:

1. Have Alice choose two bits to encode into the single quantum state
2. Have Alice encode the bits using the classical-control X and Z gates.
3. Confirm that the output state in Bob's lab is the correct state.
4. Repeat using different bit choices of Alice